TECHNIQUES FOR SIDELOBE CANCELATION

William Joseph Scott

NAVAL POSTGRADUATE SCHOOL

Monterey, California

THESIS

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by

William Joseph Scott

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William Joseph Scott
Lieutenant, United States Navy
B.S., University of New Mexico, 1963

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ABSTRACT

This thesis presents a survey of techniques which can be used to suppress sidelobes in antenna array systems. Methods applicable to fixed arrays include direct null positioning, element spacing adjustment, perturbation of complex pattern function zeroes, and pattern synthesis. Methods applicable to signal processing arrays include performance optimization, which can also be extended to permit system self-adaptation to changing noise conditions.

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I. INTRODUCTION

This thesis is intended to present a survey of techniques for the suppression of sidelobes of antenna array patterns. The use of multi-element sensor structures in seismographic, acoustic, and electromagnetic systems greatly extends the range of control over the resulting signal transmission or reception performance. A single element restricts processing capability to a single signal; a system of N elements provides the opportunity to independently manipulate N sets of operating parameters in such a way as to improve various suitable measures of system performance.

Initially we present the basic characteristics of arrays and define two fundamental performance measures; signal-to-noise ratio, SNR, and directive gain, G. Methods of sidelobe suppression applicable to fixed structures are then discussed, followed by methods which optimize performance criteria in processing systems.

It should be noted that while the methods described encompass a large portion of current theory they certainly do not exhaust the field. The capabilities of large-scale computer-controlled signal processors, the multitude of useful array geometries, and the variety of suitable performance criteria insure that this field will remain a productive area of theoretical and practical research.

II. BASIC ARRAY PRINCIPLES

An "array" may be generally defined as an arbitrary collection of sensors arranged in an arbitrary spatial pattern used to simultaneously detect the local states of an impinging field or superposition of fields. Historically, the concept of an array has been nowhere as general as this; arrays have evolved from the extension to additional numbers of elements of fixed geometrical patterns, such as the circular arrays of underwater sensors, or the linear and planar arrays of radio antennas. Theories of array behavior have shown a corresponding evolution from specific, regular algebraic forms, such as the complex polynomials of Schelkunoff [13] or the Tchebyscheff polynomials of Dolph [7], to the stochastically controlled self-adaptive processors of Mermoz [11], Widrow [17], and Griffiths [10], where the very indefiniteness of observed signals provides the means of prescribing the appropriate processing technique.

A. PATTERN FUNCTIONS

The elementary N-element linear array, Figure 1, has a total far-field radiation vector

$$V(\theta) = V_o(\theta) \sum_{n=0}^{N-1} a_n \, e^{jnkd\sin\theta} = V_o(\theta) \; S(\theta)$$

where V_o is the radiation vector of each element, a_n is the complex excitation of the n-th element, $S(\theta)$ is the space factor of the array, as defined by Ramo and Whinnery [12],

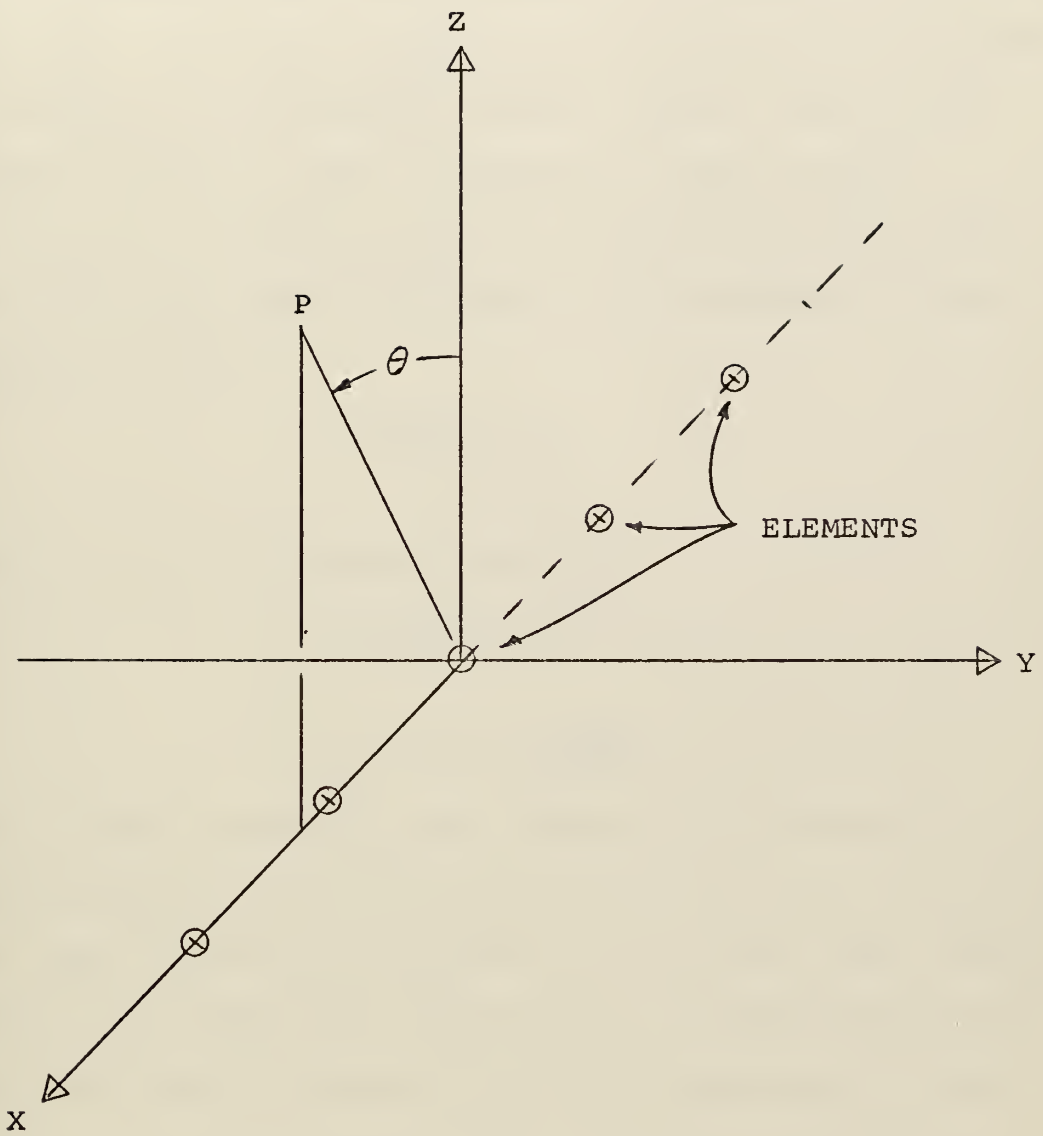

Figure 1. N-Element Linear Array

and k is the wave number. The radiation intensity is

$$K = K_o |S|^2.$$

Thus, in systems where the principle of superposition is applicable, the field patterns of an array can be arranged as a product of the element pattern times a function of angular direction. $|S|^2$ is called the array factor.

If a_n is a constant in both amplitude and phase, the pattern has a principal maximum in the X-Z plane in the direction given by

$$nkd\sin\theta = 0$$

$$\theta = 0,$$

and nulls in the directions given by

$$Nkd\sin\theta = 2n\pi, \quad n < N$$

$$\theta = \sin^{-1}\left(\frac{2n\pi}{Nkd}\right).$$

Having chosen a specific array geometry and excitation, it becomes apparent that the response of the array in directions other than the central direction is no longer under arbitrary control; an interfering signal may be located between the nulls resulting in degradation of the performance of the system.

The concept of the pattern function can be extended to two or three dimensions, using a general coordinate system as shown in Figure 2. In the case of regular planar arrays, the space factor is

$$S(\theta,\phi) = \sum_m \sum_n a_{mn} e^{\,jk(md_x \sin\theta\cos\phi + nd_y \sin\theta\sin\phi)}$$

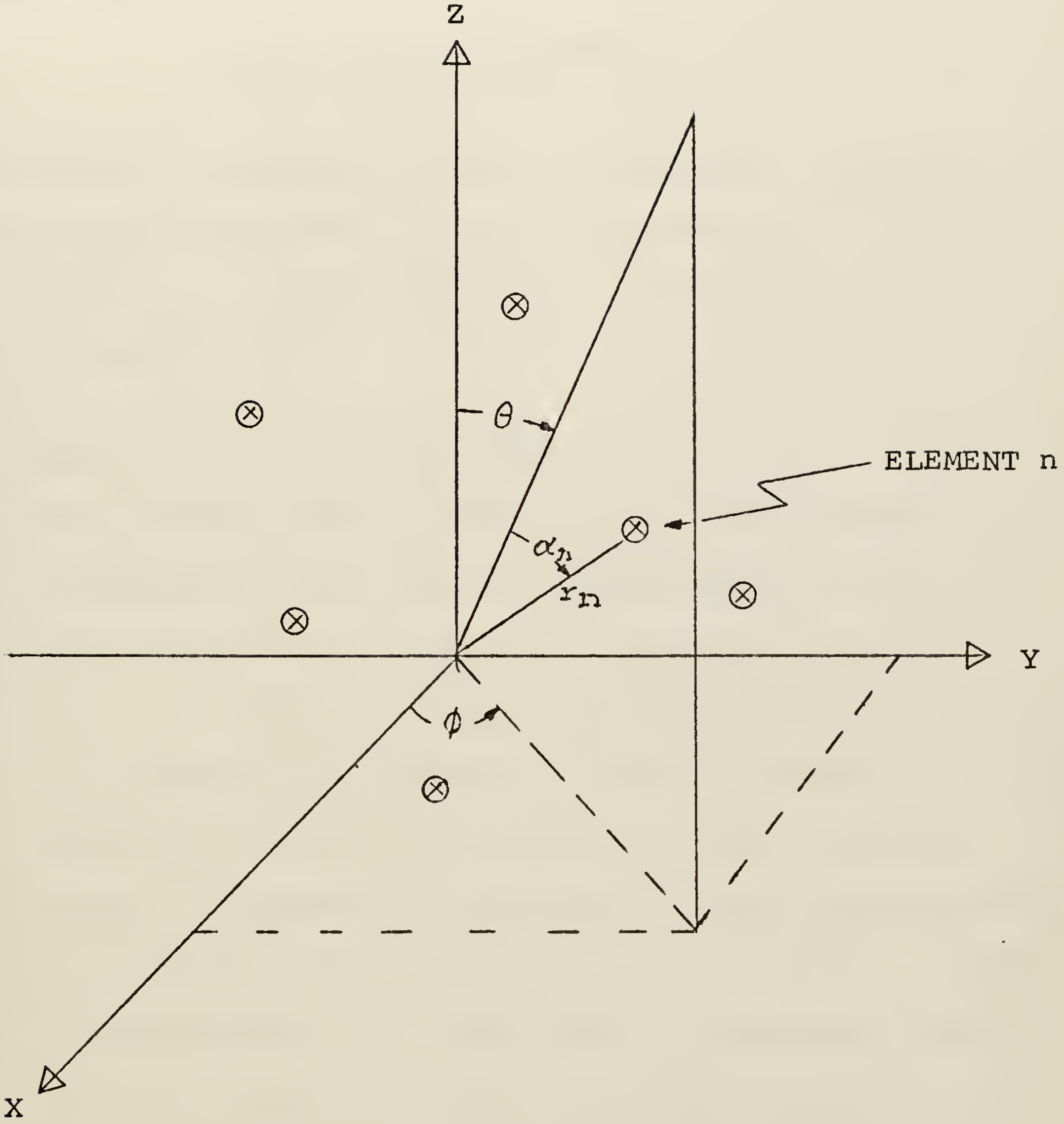

Figure 2. General Array Geometry

where (θ,ϕ) are the direction coordinates of beam orientation, and d_x and d_y are the element spacing in the X and Y directions, respectively. The general 3-dimensional array has a space factor

$$S(\theta,\phi) = \sum_n a_n e^{jkr_n \cos\alpha_n}$$

where $\cos\alpha_n = \sin\theta\sin\theta_n\cos(\phi-\phi_n) + \cos\theta\cos\theta_n$, (θ_n,ϕ_n,r_n) are the position coordinates of the n-th element and α_n is the angle between the "look direction" (θ,ϕ) and the element position vector.

B. SIGNAL-TO-NOISE RATIO AND GAIN

Signal-to-noise ratio, SNR, is a critical parameter in many receiving systems, such as echo-ranging systems and most amplitude-modulation systems. Similarly, gain, or directive gain, G, is a critical signal transmission parameter. In considering problems of antenna sidelobes, it has been suggested that off-axis signals may cause unwanted interference. Conversely, adjustment of the antenna pattern to lower sidelobes may degrade on-axis performance. Techniques of optimization to be described will maximize SNR or G, thus achieving a balanced compromise between sidelobe level and main-beam degradation. In general, SNR is defined as

$$\mathrm{SNR} = \frac{\text{output signal power without noise}}{\text{output noise power without signal}}$$

$$= P_s/P_o.$$

This parameter is meaningless without appropriate definition of the processing system. A general system is shown in

Figure 3. The output signal is

$$s_o = \sum_m s(t+\tau_m) * h_m(t)$$

where the incident signal is assumed to have the same structure as a function of time at all sensors, τ_m is the time delay of the signal at the m-th element, h_m is the impulse response of the m-th element, and * represents the convolution operation. The signal power is then

$$P_s = \sum_m \sum_n \int_{-\infty}^{\infty} W_s(\omega) H_m(\omega) H_n^*(\omega) e^{j\omega(\tau_m - \tau_n)} d\omega/2\pi$$

where the computation has been expressed in the frequency domain with signal power spectral density $W_s(\omega)$, filter transfer function $H(\omega)$, and radian frequency variable ω. The noise output is

$$n_o = \sum_m n(t + \tau_m) * h_m(t),$$

so the resulting noise power is

$$P_o = \sum_m \sum_n \int_{-\infty}^{\infty} W_o(\omega) H_m(\omega) H_n^*(\omega) e^{j\omega(\tau_m - \tau_n)} d\omega/2\pi.$$

An ultimate objective is to manipulate the gain and phase shift of the filters in a frequency-independent manner so the filter response can be represented as

$$H_m(\omega) = K_m e^{j\phi_m} H(\omega)$$

with $H(\omega)$ the frequency-independent factor. Then,

$$P_s = \sum_m \sum_n K_m K_n e^{j(\phi_m - \phi_n)} \int_{-\infty}^{\infty} W_s(\omega) |H(\omega)|^2 e^{j\omega(\tau_m - \tau_n)} d\omega/2\pi$$

and

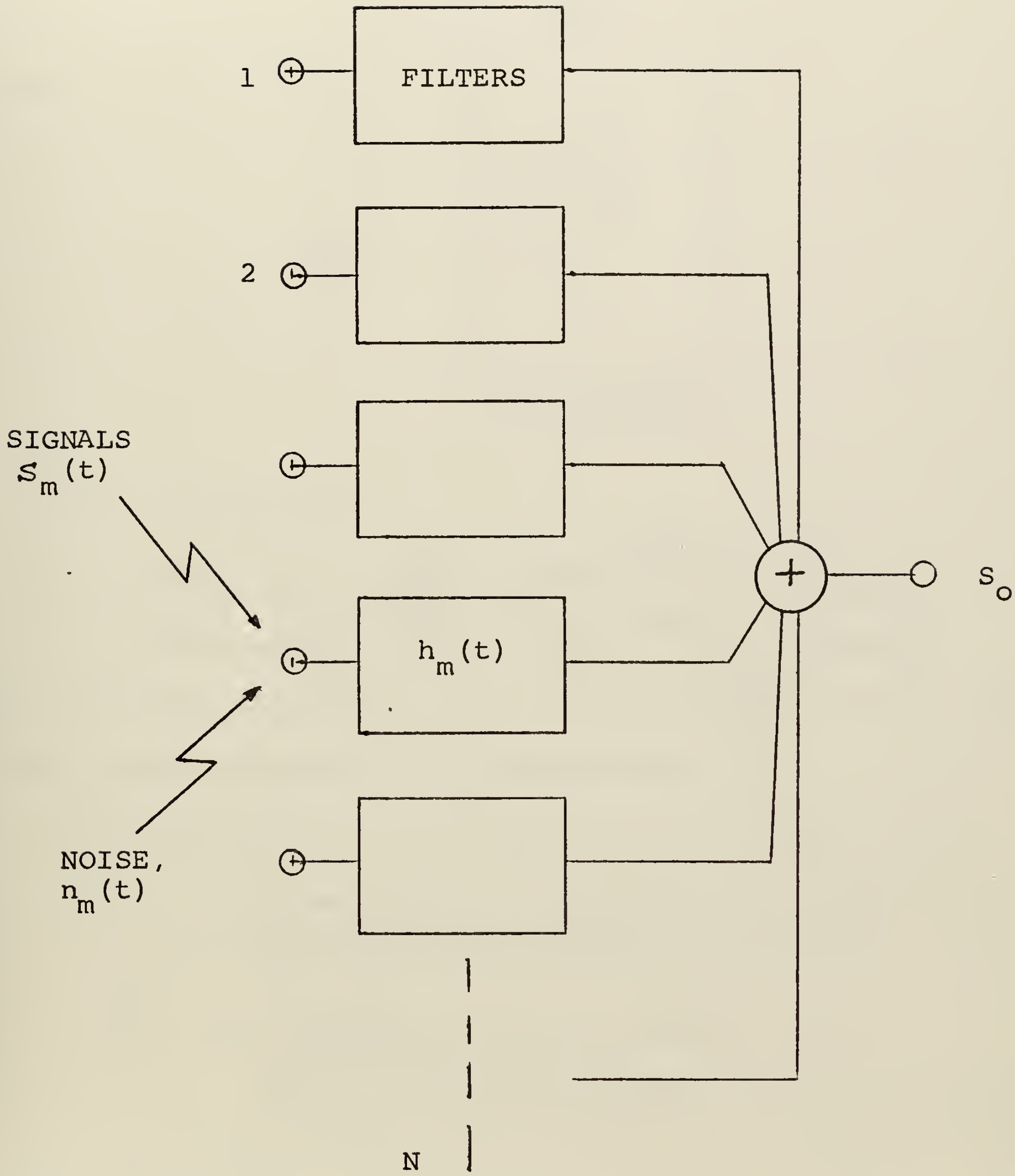

Figure 3. General N-Channel Linear Processor

$$P_o = \sum_m \sum_n K_m K_n e^{j(\phi_m - \phi_n)} \int_{-\infty}^{\infty} W_o(\omega) |H(\omega)|^2 e^{j\omega(\tau_m - \tau_n)} d\omega/2\pi$$

Finally, SNR may be expressed in matrix form as

$$SNR = \frac{K^\dagger G K}{K^\dagger Y K}$$

where

$$K = \begin{bmatrix} K_0 e^{j\phi_0} \\ K_1 e^{j\phi_1} \\ \cdot \\ \cdot \\ \cdot \\ K_{M-1} e^{j\phi_{M-1}} \end{bmatrix}$$

$$K^\dagger = \text{Adjoint } [K]$$

$$G = [g_{mn}],\ g_{mn} = \int_{-\infty}^{\infty} W_s(\omega) |H(\omega)|^2 e^{j\omega(\tau_m - \tau_n)} d\omega/2\pi$$

$$Y = [y_{mn}],\ y_{mn} = \int_{-\infty}^{\infty} W_o(\omega) |H(\omega)|^2 e^{j\omega(\tau_m - \tau_n)} d\omega/2\pi$$

The resulting formula, a ratio of two quadratic forms, provides unique properties for optimization.

Directive gain, G, as defined by Silver [15] is

$$G = \frac{\text{power density in direction } (\theta_o, \phi_o)}{\text{average power density}}$$

$$= \frac{P(\theta_o, \phi_o)}{\frac{1}{4\pi} \int_0^{2\pi} \int_0^{\pi} P(\theta, \phi) \sin\theta d\theta d\phi}$$

Since, for arrays,

$$P(\theta, \phi) = P_o(\theta, \phi) \ |S(\theta, \phi)|^2$$

where $P_o(\theta, \phi)$ is the element power pattern and $S(\theta, \phi)$ is the space factor of the array, the gain becomes

$$G = \frac{P(\theta_o,\phi_o)}{\frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi} P_o(\theta,\phi)\ |S(\theta,\phi)|^2 \sin\theta d\theta d\phi}$$

The array factor is

$$|S(\theta,\phi)|^2 = \sum_n\sum_m a_n a_m e^{jk(r_n\cos\alpha_n - r_m\cos\alpha_m)}$$

where the a_n now represent the weighting factors and linear filter of the receiving system,

$$a_n = K_n e^{j\phi_n} H(\omega)$$

Also, the transmitted power density in direction (θ_o,ϕ_o) is

$$P(\theta_o,\phi_o) = P_o(\theta_o,\phi_o)|S(\theta_o,\phi_o)|^2$$

so the gain becomes (Cheng [4])

$$G = \frac{P_o(\theta_o,\phi_o)|S(\theta_o,\phi_o)|^2}{\frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi} P_o(\theta,\phi)\ |S(\theta,\phi)|^2 \sin\theta d\theta d\phi}$$

$$= \frac{P_o(\theta_o,\phi_o)\sum_n\sum_m a_n a_m e^{jk(r_n\cos\alpha_{no} - r_m\cos\alpha_{mo})}}{\frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi} P_o(\theta,\phi)\sum_n\sum_m a_n a_m e^{jk(r_n\cos\alpha_n - r_m\cos\alpha_m)}\sin\theta d\theta d\phi}$$

$$= \frac{K^\dagger AK}{K^\dagger BK}.$$

This is, again, a ratio of quadratic forms, with K the vector of element weights, and

$$B = [b_{mn}],\ b_{mn} = \frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi} P_o(\theta,\phi)e^{jk(r_n\cos\alpha_n - r_m\cos\alpha_m)}.$$

$$\sin\theta d\theta d\phi$$

$$A = [a_{mn}] = DD^{\dagger}$$

$$D = \begin{bmatrix} e^{jkr_o \cos\alpha_{oo}} \\ e^{jkr_1 \cos\alpha_{10}} \\ \vdots \\ e^{jkr_{M-1} \cos\alpha_{(M-1)0}} \end{bmatrix}$$

C. SIDELOBE SUPPRESSION PRINCIPLES

The most elementary technique of interference elimination is to position a null in the interfering direction. This method is restricted to situations in which all other noises are insignificant and where appropriate care can be taken in system alignment to allow for the sharpness of nulls. Positioning a null means adjusting the relative element phases so that complete cancelation occurs in the null direction. Since maximum signal reception or radiation requires a different phasing arrangement, the technique leads to sacrifice in performance. The suppression of wide sectors of noise generally requires more than null positioning. Here it is desirable to specify some acceptable level of gain suppression in the sector. This result can be achieved by adjustment of both element gain and phase. An artificial method of gain adjustment involves modifying the positions of the elements in a regular array structure. The complex polynomial technique of Schelkunoff permits both direct positioning of zeroes corresponding to desired nulls, and

clustering of zeroes to reduce sidelobe levels over an angular sector.

Optimization processes permit use of the total array environment to adjust the element filters to the optimal values. Thus, background noise, sectoral noise, and narrow-beam interference may all be considered. In addition, the observation of the noise environment at the sensors can provide information which can be used in a real-time sense to readjust the filters, thus permitting the system to continually adapt itself to the interference environment.

III. FIXED ARRAY SYSTEMS

Fixed array systems are those in which the parameters of the array, such as element position, gain, and phase, are determined and the array is constructed according to the resulting specifications. Methods of sidelobe cancelation in this situation include direct null positioning, spacing adjustment, perturbation of complex pattern function zeroes, and pattern synthesis.

A. DIRECT NULL POSITIONING

Schelkunoff [13] showed that pattern functions for linear arrays can be represented by complex polynomials on the unit circle

$$f(Z) = \sum_{m=0}^{M-1} a_m Z^m, \quad Z = e^{(jkd\sin\theta - \phi)}$$

where ϕ is a constant phase shift per element and a_m is the complex weighting coefficient for the m-th element. Zeroes of f(Z) lie on the unit circle and define the positions of nulls of the pattern. An M-element array will therefore have M-1 roots of f(Z), determined by the a_m, and conversely, the a_m can be selected to independently position M-1 nulls. The resulting design will require individual element feeds capable of both amplitude and phase control.

The concept of virtual couplets provides a null positioning system described by Davies [6]. An M-element array has a pattern which can be interpreted by the pattern

multiplication principle as the product of a couplet, or two-element pattern, and an M-1 element space factor. This reasoning can be extended to its ultimate reduction of the array to a collection of couplets. The couplets must have the same relative phasing at each stage. The resulting structure for a four-element array is shown in Figure 4. In this case, there are three independently-controlled phase shifts, which permit control of three pattern nulls.

B. SPACING ADJUSTMENT

This method, due to Strait and Cheng [16] uses adjustments in the spacing of the linear array elements to produce results similar to amplitude weighting of the previous section. A particular advantage, however, is that element gains can now be fixed at values satisfying other requirements, such as fixed-power matched drivers. For a linear array of N elements with N even and cophasal excitation, the space factor is proportional to

$$S = \sum_{n=1}^{N/2} I_n \cos\left[\left(\frac{2n-1}{2}\right) kd\sin\theta\right]$$

For a variation in spacing v,

$$S_v = \sum_{n=1}^{N/2} I_n \cos\left[\left(\frac{2n-1}{2}\right) k(d+V)\sin\theta\right]$$

$$= \sum_{n=1}^{N/2} I_n \cos\left[\left(\frac{2n-1}{2}\right) kd\sin\theta + \left(\frac{2n-1}{2}\right) kv\sin\theta\right]$$

$$= \sum_{n} I_n \cos\left[\left(\frac{2n-1}{2}\right) + c_n\right]\Psi$$

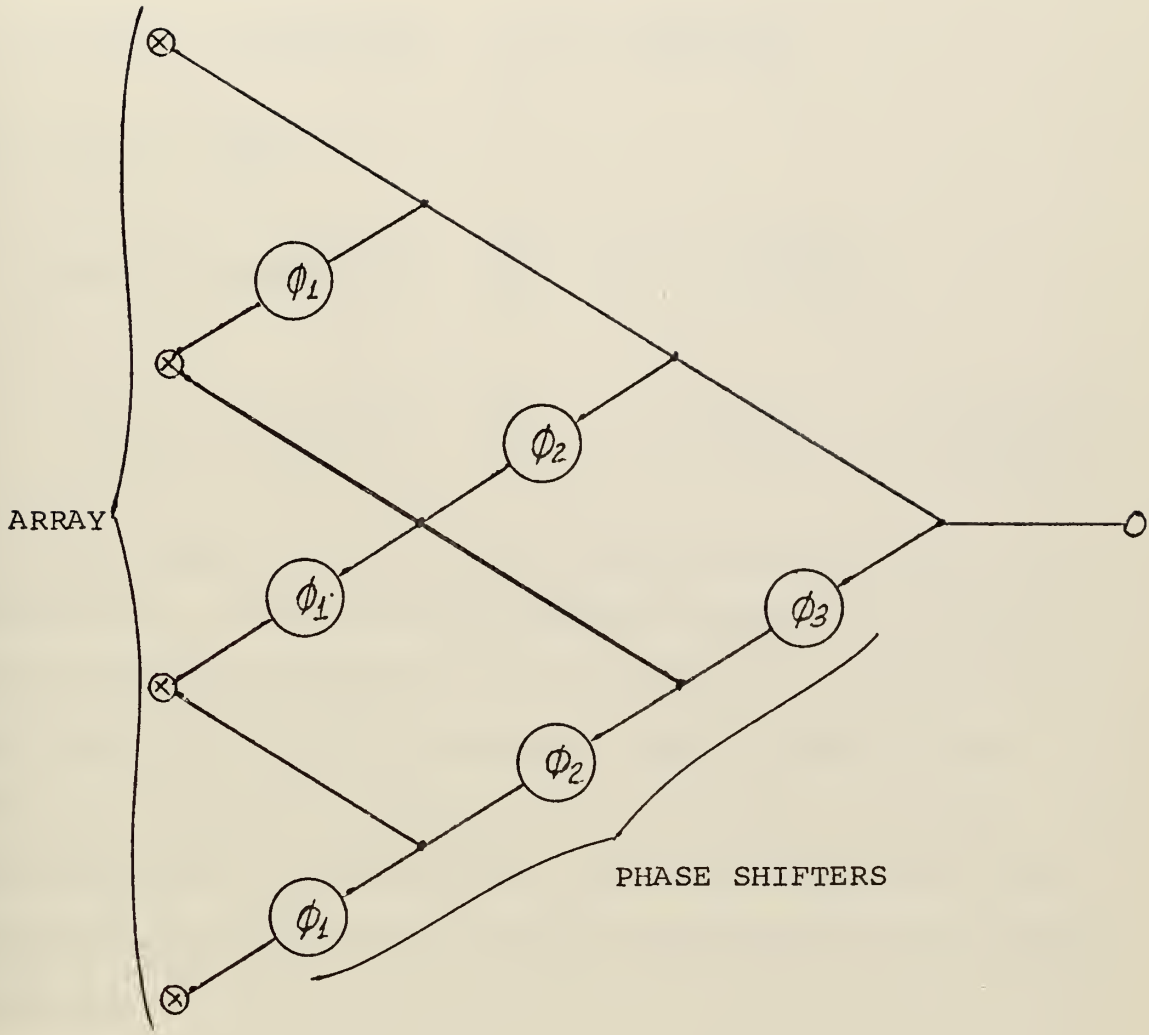

Figure 4. Null-Positioning Circuit

where $\Psi = kd\sin\theta$,

$$c_n = \left(\frac{2n-1}{2}\right)\frac{v}{d}.$$

For $c_n\Psi$ sufficiently small,

$$S_v \approx \sum I_n \left[\cos\left(\frac{2n-1}{2}\right)\Psi - c_n\Psi \sin\Psi\left(\frac{2n-1}{2}\right)\right].$$

Since $\sin\Psi \approx \frac{\pi}{4}\Psi$,

$$S_v \approx \sum I_n \left[\cos\left(\frac{2n-1}{2}\right)\Psi - \frac{4c_n}{\pi}\sin\Psi\sin\left(\frac{2n-1}{2}\right)\Psi\right]$$

$$\approx \sum I_n \left[\cos\left(\frac{2n-1}{2}\right)\Psi + \frac{2c_n}{\pi}\left\{\cos\left(\frac{2n+1}{2}\right)\Psi - \cos\left(\frac{2n-3}{2}\right)\Psi\right\}\right]$$

It is then apparent that, given a desired space factor as a cosine series with arbitrary element weights, the series S can be equated term by term and the factors c_n computed in such a way as to hold I_n constant. Such a result would permit the design of transmitters for optimum, constant operating power characteristics, and would also avoid the complications of individual element amplification and phase weighting.

C. PERTURBATION OF ZEROES

This approach involves making small adjustments in the locations of zeroes of the complex array polynomial. A similar technique has been used by Baker [1], to modify features of another otherwise suitable pattern. Since the space factor is

$$S(\theta) = \prod_{n=1}^{N-1} (Z - Z_n)$$

the array factor is

$$P(\theta) = \left| S(\theta) \right|^2 = \prod_{n=1}^{N-1} 2e^{-a_n} \left[\cosh a_n - \cos(\Psi - b_n) \right]$$

where $Z = e^{j\Psi}$, $\Psi = kd(\sin\theta) - \phi$,

and $Z_n = e^{-a_n + jb_n}$ are the roots of the polynomial. Since it is assumed that a_n is not necessarily zero, the root at Z_n is not necessarily a null of the pattern. The basic idea is to start with a known, approximately satisfactory pattern, $P_o(\theta)$, and to construct a desired modification of the pattern, as shown in Figure 5. Variations may now be made in a_n and b_n to minimize the excess amplitude of the sidelobes in the sector of interest, or to minimize the mean-squared-difference between the desired pattern and the actual pattern. Normally, it would be necessary to define the reference pattern over the entire range from $\theta = 0$ to $\theta = \pi$. Otherwise, adjustments resulting from differences in a given sector may cause reduction of performance in the main beam.

D. PATTERN SYNTHESIS

This procedure is also based on the theory of Schelkunoff. A desired space factor may be defined by an arbitrary function, $F(\theta)$, over the range $\theta = 0$ to $\theta = \pi$, or its equivalent function of Ψ, $f(\Psi)$, where

$$\Psi = kd\sin\theta - \phi$$

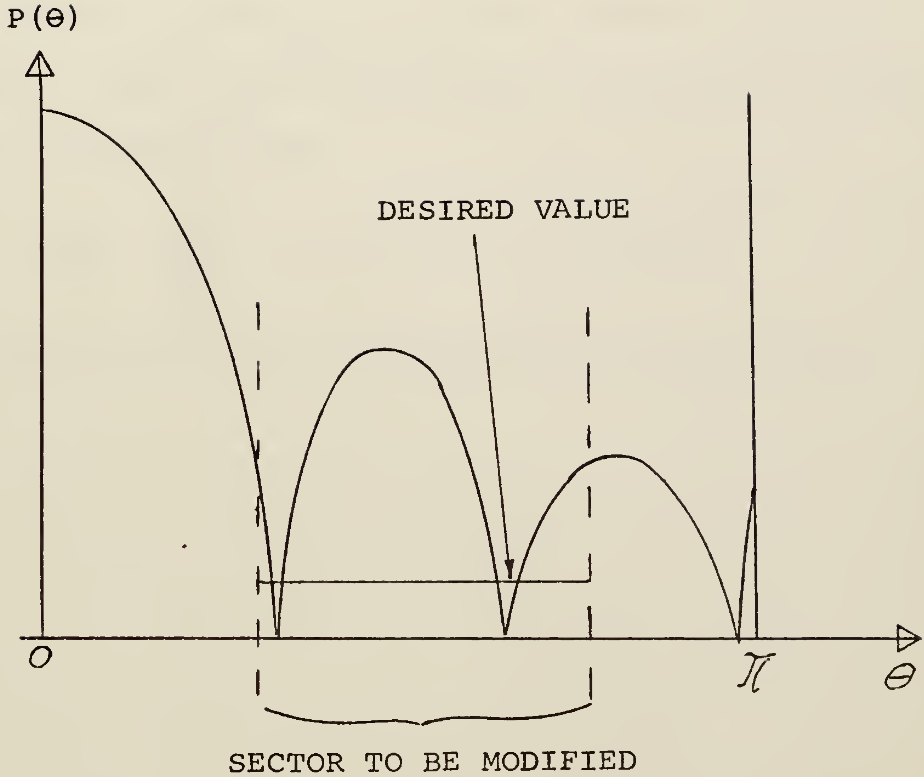

Figure 5. Pattern Modification

If the array has an odd number of elements and the coefficients equidistant from the ends of the polynomial are required to be conjugates, the space factor becomes

$$S(\psi) = \sum_{n=0}^{N} (A_n \cos n\psi + B_n \sin n\psi)$$

where the unknown A_n and B_n are related to the polynomial coefficients. Then, the desired space factor may be expanded as a Fourier series so that

$$f(\psi) = \sum_{n=0}^{\infty} (P_n \cos n\psi + Q_n \sin n\psi)$$

and the coefficients of the two series can be equated.

IV. LINEAR PROCESSING ARRAYS

Two types of signal processing systems utilize individual filtering, or weighting, of each array element to optimize a performance characteristic of the array. In one method the signal to noise ratio, SNR, is maximized. Two general approaches to SNR maximization have been considered by Mermoz [11] using N-channel matched filters, and by Cheng [3,4] using the properties of Hermitian matrices. In the second method the directive gain of the array is optimized, subject to constraints which may include specification of the desired location of pattern nulls. The objective of SNR maximization is to obtain a set of filters which result in maximizing knowledge of the presence of a signal when the signal and noise are additive. The maximization techniques require knowledge of the structure of noise in the surrounding environment; for the matched filter approach, the noise must be observable by measurement at the array elements in order to determine its statistical properties. This has the advantage of permitting adjustment of the filters on the basis of the noise measurement if the noise environment changes; however, it also requires a capability to process direct noise information separately from filtered signal plus noise. The matrix approach is equivalent to the matched filter approach in receiving systems;it permits constraints on null locations to be included in the optimization.

A. SNR OPTIMIZATION

It was shown that the signal-to-noise ratio can be represented by

$$SNR = \frac{K^{\dagger}GK}{K^{\dagger}YK}$$

where K is the vector of element weights and G and Y are square matrices relating the relative element time delays and the filter, signal, and noise power spectral densities. Cheng [3] has used the resulting function to show that if G can be represented by

$$G = F_o F_o^{\dagger},$$

where

$$F_o = \begin{bmatrix} e^{-jkr_o \cos\alpha_{oo}} \\ e^{-jkr_1 \cos\alpha_{01}} \\ \vdots \\ e^{-jkr_{M-1} \cos\alpha_{(M-1)}} \end{bmatrix}$$

then the maximum SNR is

$$SNR_{max} = F_o^{\dagger} Y^{-1} F_o$$

and the maximizing filter weights are

$$K_M = Y^{-1} F_o.$$

The vector F_o consists of the relative phase functions of the signal at each element of the array. Normalizations have been assumed so that the results are valid only within a constant factor. Mermoz [11] has used the correlation

properties of signals and noises at the various elements to arrive at the result that the optimizing filter, $H_n(\omega)$, is given by

$$\sum_n C_{mn}(\omega)\, H_n(\omega) = S_m^*(\omega)$$

where C_{mn} is the noise cross-power spectral density of the m and n-th elements and S_m is the conjugate of the signal spectrum. It has been assumed that the signal input to the filters has been delayed to compensate for the relative delay of the signals; this operation was previously performed by the vector F_o. The result, then, for optimal performance is that

a) The signal should be delayed by a combination of signal time delay and noise correlation;

b) The signal should be attenuated in proportion to the noise correlation;

c) The result should be linearly filtered by a matched filter, $H(\omega) = S^*(\omega)$.

Combining these results, the complete filter system is given by

$$H_m(\omega) = (C^{-1}F_o)_m \; S_m^*$$

For the M-element system, F_o provides the M synchronizing delays, C is the MxM matrix of noise correlations (in the frequency domain), and H is the Mx1 vector of channel filters. S^* is 1xM. If the noise environment is a single coherent signal (jamming) from a certain direction, the optimization problem reduces to adding the element outputs

in such a way as to maintain synchronization of the desired signal, while achieving maximum cancelation of the interfering signal. A two-element monochromatic situation as shown in Figure 6 illustrates the application of these filtering concepts. For the phase and time-delay reference at element 1, the relative signal and noise delays at element 2 are

$$\tau_s = \frac{1}{c} (d \sin \theta_s)$$

$$\tau_N = \frac{1}{c} (d \sin \theta_N)$$

If filters H_1 and H_2 are assumed to consist of a signal-synchronizing delay followed by an inverse noise covariance filter, C^{-1}, then

$$\begin{bmatrix} H_1 \\ H_2 \end{bmatrix} = \begin{bmatrix} 1 \\ e^{j\omega\tau_s} \end{bmatrix} \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} C_{11}^{-1} & C_{12}^{-1} \\ C_{12}^{*-1} & C_{22}^{-1} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

In this formulation, amplitude normalization has been assumed. Since the noise is coherent and monochromatic,

$$C_{11}^{-1} = C_{22}^{-1} = 1,$$

$$C_{12}^{-1} = C_{21}^{*-1} + e^{j\omega(\tau_N - \tau_s)}.$$

Accordingly, with some algebra,

$$\begin{bmatrix} H_1 \\ H_2 \end{bmatrix} = \begin{bmatrix} 1-e^{-j\omega(\tau_N - \tau_s)} \\ e^{j\omega\tau_s} \left\{1-e^{j\omega(\tau_N - \tau_s)}\right\} \end{bmatrix}$$

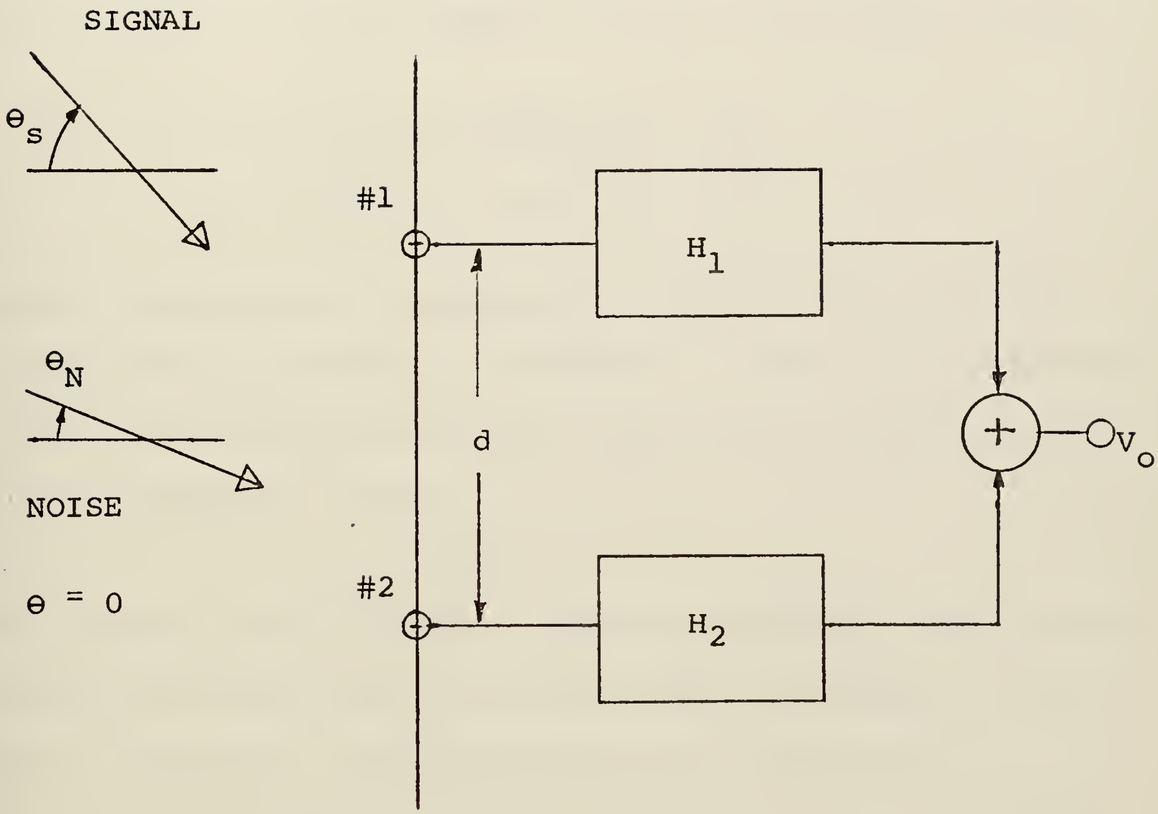

Figure 6. 2-Element Filter System

For signal and noise inputs A and N at element 1 and $Ae^{j\omega\tau_s}$ and $Ne^{j\omega\tau_n}$ at element 2, the filter outputs are

$$V_1 = (A+N)\left[1 - e^{-j\omega(\tau_N - \tau_s)}\right]$$

$$V_2 = (Ae^{-j\omega\tau_s} + Ne^{-j\omega\tau_N})\ (e^{j\omega\tau_s})\left[1 - e^{j\omega(\tau_N - \tau_s)}\right].$$

Addition of the filter outputs results in the final output,

$$V_o = V_1 + V_2 = 2A - 2A\cos\omega(\tau_N - \tau_s)$$

$$= 2A\ [1 - \cos\omega(\tau_N - \tau_s)].$$

Certain comments are appropriate concerning the procedure followed before evaluating the result. Under the assumption of coherent monochromatic noise, the covariance matrix, C, is not invertible, since

$$\det C = 0.$$

This implies that an infinite SNR is obtainable, which means, as stated by Mermoz [11], that the noise is completely cancelable. However, the construction of phase-shifting filters in the desired structure can still be carried out. The procedure also assumes that relative attenuation between the sensors is negligible, which would be approximately true in practice. The resulting output signal contains no noise component; therefore the process of cancelation is valid regardless of the angles involved. However, the signal amplitude is reduced by delay of the noise relative to the signal. If the noise and signal are from the same direction, $\tau_N = \tau_s$, and the output is zero. In fact, due to the broadness of the cosine function near

$\tau_N - \tau_s = 0$, resolution of the signal and noise will be difficult over a large angular range. The resulting output signal, V_o, should be passed through a matched filter in the general case; for monochromatic signals, this would be a high-Q resonant filter.

B. GAIN OPTIMIZATION

Maximization of directive gain, G, is accomplished using the same characteristic of quadratic forms as used for SNR. Thus, if

$$G = \frac{K^{\dagger}AK}{K^{\dagger}BK},$$

then

$$G_{max} = D^{\dagger}B^{-1}D$$

and

$$K_m = B^{-1}D,$$

where K_m is the vector of maximizing filter weights, A, B, and D are as previously defined. Since the elements of B are

$$b_{mn} = \frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi} P_o(\theta,\phi)\, e^{jk(r_n \cos\alpha_n - r_m \cos\alpha_m)} \sin\theta\, d\theta\, d\phi$$

it is apparent that maximizing G is equivalent to maximizing SNR under the assumptions of monochromatic operation with isotropic elements in omnidirectional noise.

It may be desirable to utilize constraints in the optimization of SNR or G. In the case of transmitting systems, radiation in a certain direction may cause excessive interference; for receivers, a fixed noise source may be more

easily handled by simply requiring a null in its (

rather than designing the correlation processes o

needed. Cheng [3], and Drane and McIlvenna [9] h

lined procedures to accomplish the constrained ma

based on transformation of the A and B matrices to an abridged form which includes the effect of the constraining relations.

C. ADAPTIVE SYSTEMS

The signal processing systems previously described can be designed for a static mode of operation in which the filter weights are predetermined. The array may accordingly be adjusted for a fixed main beam direction and fixed noise distribution or for scanning beams. The full capabilities of the correlation principles used are achieved when the design is extended to time-varying situations in which the sidelobe structure depends on the orientation of the main beam or the noise environment is subject to random variation. In such cases, the correlation measurements can be made on a real time basis, thereby keeping the system operation in nearly optimal condition. The time varying characteristics are usually assumed to vary slowly; otherwise, the statistical model of the noise may be over-complicated. The main problem in developing a self-adaptive arrangement is construction of the correlation functions. One general scheme for control of the filters is shown in Figure 7. Operation of this system, as described by Brennan, et. al. [2], is defined by

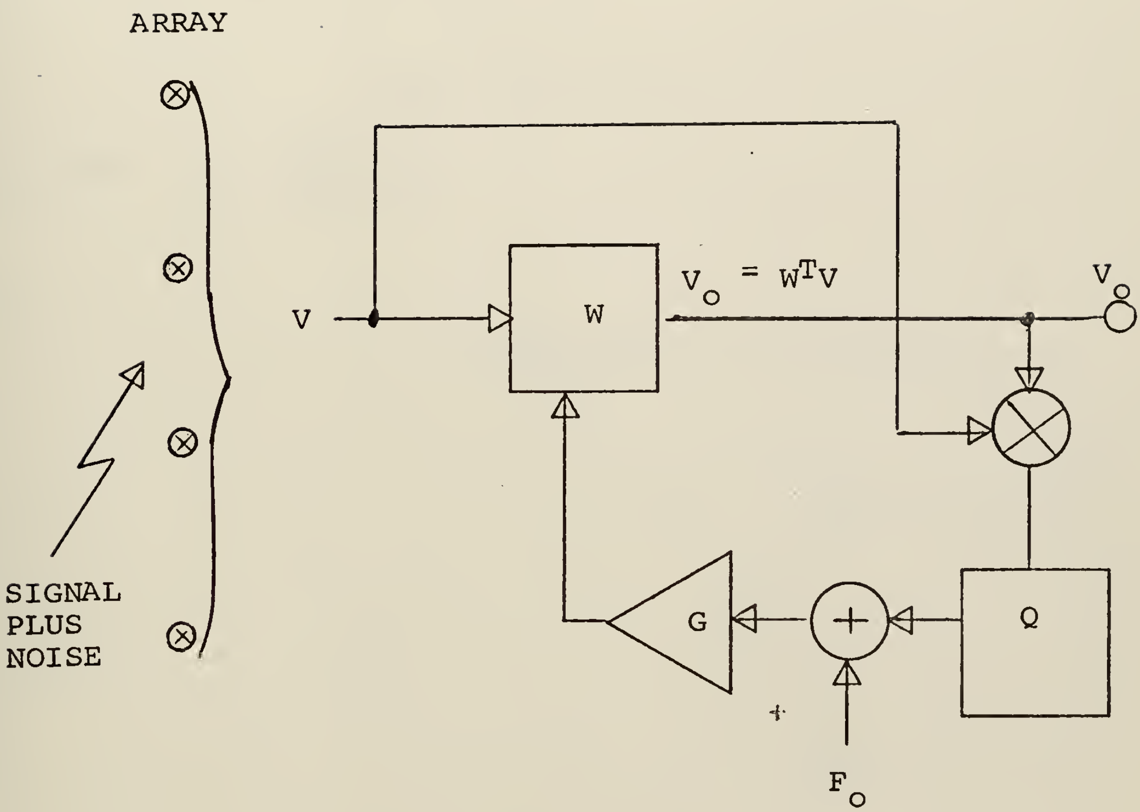

Figure 7. Adaptive System for SNR Maximization

$$W = G(F_o - Z)$$

$$Z = Q(V^* W^T V)$$

where F_o is the steering vector and Q is the Laplace transform of a linear filter, and W is the vector of weights. The desired covariance matrix is

$$m_{ij} = E\left\{v_i^* v_j\right\}.$$

But $W^T V = V_o = \sum_{i=1}^{N} W_i V_i$

so

$$E\left\{V^* W^T V\right\} = E\left\{V^* \sum_{i=1}^{N} w_i v_i\right\}$$

$$= E\left\{\begin{bmatrix} \sum_{i=1}^{N} v_1^* w_i v_i \\ \vdots \\ \sum_{i=1}^{N} v_N^* w_i v_i \end{bmatrix}\right\}$$

Also,

$$MW = \begin{bmatrix} \sum_{i=1}^{N} m_{1i} w_i \\ \vdots \\ \sum_{i=1}^{N} m_{Ni} w_i \end{bmatrix}$$

and since

$$m_{ni} = E\left\{v_m^* v_i\right\}$$

it is clear that

$$E\left\{V^* W^T V\right\} = MW.$$

The weights are thus seen to be controlled by the difference between the steering vector and a filtered function of the product of covariance and weights. In the first-order case,

$$Q = \frac{1}{\tau s + 1}$$

and in the second order case

$$Q = \frac{1}{\tau_1 \tau_2 s^2 + (\tau_1 + \tau_2)\, s + 1}$$

Here τ represents the eigenvalues of the smoothing process, Q, not to be confused with relative signal and noise delays. This form of filter control leads to the optimal weights which maximize SNR. Widrow [17] has considered another procedure shown in Figure 8 in which the system adapts to match a desired signal by minimizing the mean squared error. The pilot signal can be injected internally, or added to the external signal and noise field. Doyle's [8] comparison of the SNR-maximization and signal-matching systems indicates that the pilot-signal technique requires high power to achieve adequate "lock-on".

D. SPATIAL SIGNAL DISTRIBUTIONS

The systems previously described have modeled signals and noise as observable functions of time for which time-correlations can be constructed; the external, spatial properties of the signals have been assumed to be restricted to time delays which are explicitly related to spatial

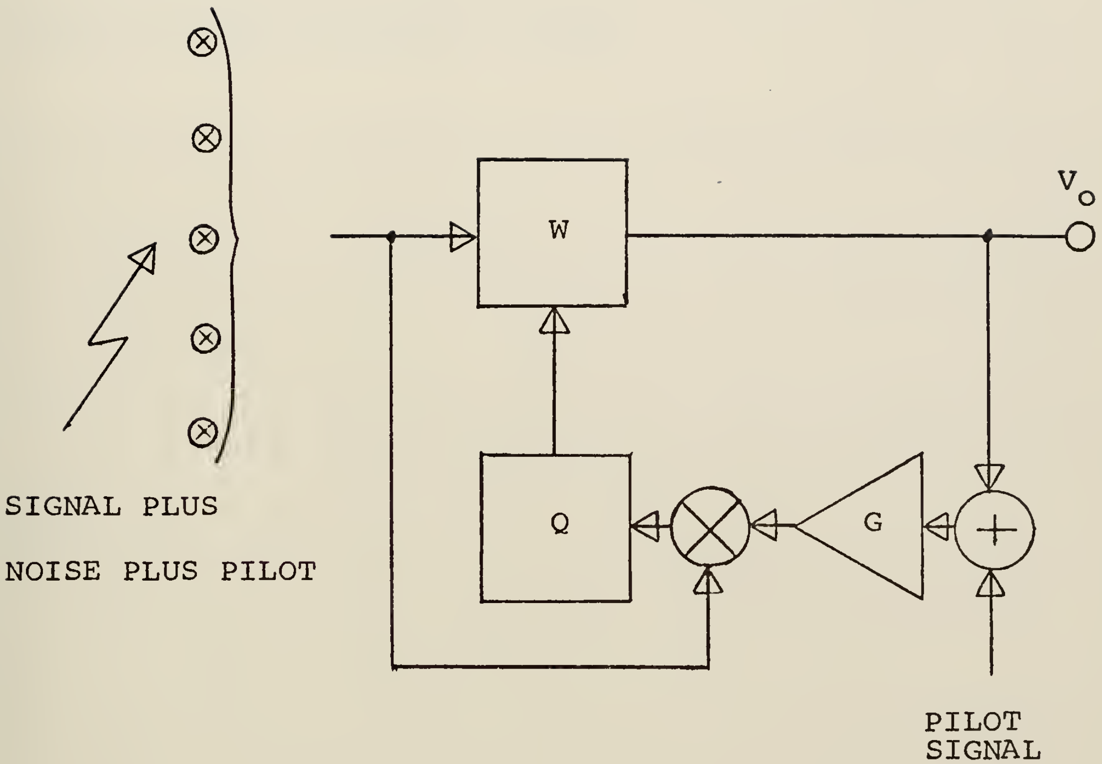

Figure 8. Adaptive Signal-Matching System

direction. These assumptions are not overly restrictive in normal applications; however, they may be eliminated by application of a more general theory of space-time cross-correlation functions. Such a theory has been suggested by Childers and Reed [5]. Use of these concepts has been considered by Cheng and Tseng [4] in the formulation of signal-to-noise ratio for arrays.

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13. ABSTRACT

This thesis presents a survey of techniques which can be used to suppress sidelobes in antenna array systems. Methods applicable to fixed arrays include direct null positioning, element spacing adjustment, perturbation of complex pattern function zeroes, and pattern synthesis. Methods applicable to signal processing arrays include performance optimization, which can also be extended to permit system self-adaptation to changing noise conditions.

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